Third International Conference on Inverse Design Concepts and Optimization in Engineering Sciences (ICIDES-III), Editor: G.S. Dulikravich, Washington D.C., October 23-25, 1991.

INVERSE PROBLEMS IN THE DESIGN, MODELING AND TESTING OF ENGINEERING SYSTEMS

N92-13966

Dr. Oleg M. Alifanov, Professor and Chairman of Department of Space System Engineering and Design, Dean of the College of Cosmonautics, Moscow Aviation Institute, 4, Volokolamskoye Sh., Moscow, 125871, USSR

p. 18

ABSTRACT

Formulations, classification, areas of application, and approaches to solving different inverse problems are considered for design of structures, modeling, and experimental data processing. Problems in the practical implementation of theoretical-experimental methods based on solving inverse problems are analyzed in application to identification of mathematical models of physical processes, input data preparation for design parameter optimization, design parameter optimization itself, model experiments, large-scale tests, and real tests of engineering systems. This methodology provides an opportunity to improve the quality of investigations and to accelerate realization of research achievement.

INTRODUCTION

The process of design and testing of a new complex technical object can be arbitrarily divided into a number of steps and sections (Fig. 1). Each of them is very important and essential. If the problems are posed correctly and their solutions are accurate at each step then the developed engineering system will be effective and reliable. Very often structures of today vehicles work in extreme modes, on the limit of structural materials capacity. That is why any mistake made on any of the stages of design and experimental development could result in a catastrophe comparable to those of Chernobyl or Challenger.

Operational conditions of technical equipment in many industries become more and more sophisticated and severe. At the same time, the requirements for reliability and service life as well as effective technological decisions also grow. Therefore, we need not only to improve old, traditional methods of research, design and testing of structures but also to develop altogether new, more perfect ones. To these new methods we can refer those based of solution of inverse problems. The latest 15-20 years witness permanent growth of interest to them. How can we explain it? First of all, this approach made it possible to consider real phenomena taking into account non-linearity and non-stationarity of physical processes characterising today engineering systems. This is a very important point, since the above mentioned phenomena become determinating when operational conditions of the vehicles approach criticality. Conventional classical methods can hardly cope with these difficulties.

The chief advantage of the inverse problems methods is that they enable us to conduct experimental studies under conditions as close as possible to real ones or to study the engineering systems directly. Also, such approach enhances the informative value of these studies, accelerating the experimental works as compared to the traditional methods, and reducing their cost. Besides

Third International Conference on Inverse Design Concepts and Optimization in Engineering Sciences
(ICIDES-III). Editor: G.S. Dulikravich, Washington D.C., October 23-25, 1991.

taking into account non-stationarity and non-linearity, inverse problems methods provide an opportunity to analyse account multidimensionality and interdependency of physical processes, indirect measurements, and real scale of time.

All these advantages and possibilities of inverse problems are of special importance for aerospace and rocket technology. Therefore some of the first formulation and solution processes for the inverse problems, in particular, the inverse heat transfer problems, appeared in this area of application.

GENERAL FORMULATION OF INVERSE PROBLEMS AND THEIR CLASSIFICATION

All phenomena in nature are characterized by some cause-and-effect relationships, and it is possible in the construction of mathematical models of physical processes to designate quantities that are causal characteristics of the process and quantities that are resultant characteristics.

Accordingly, all problems can be classified into two types. In the first, they involve study of the effect on the basis of given causes. These are direct problems. In the second - study of the causes on the basis of specified effects. These are inverse problems. Inverse problems have one common attribute in contrast to the case of direct problems. Their formulations cannot be reproduced in a real experiment. It is not possible to reverse the cause-and-effect relation physically, instead of mathematically. For example, it is impossible to reverse the course of a heat transfer process or to change the course of time. Therefore, in mathematical formalization, this property is manifested in incorrect mathematical conditioning and must be taken into account in the development of solution methods and in applying them in practice. When formulating general statements of inverse problems and choosing the main classes of them, the statements of direct problems are supposed to be known. Each direct problem (within the framework of an accepted mathematical model) can be compared with a certain set of inverse problems. All inverse problems can be divided into three classes on the basis of the general objective: inverse problems that arise in the diagnostics and identification of physical processes; inverse problems that arise in the design of engineering products; inverse problems that arise in the control of processes and products.

Inverse problems of the first class usually involve experimental studies. In these cases it is necessary to reconstruct causal characteristics on the basis of certain measured "output" effect characteristics. These problems are primary, both with respect to direct problems and with respect to the other two classes of inverse problems, since they are connected with construction of mathematical models and determination of different characteristics of the models.

Inverse problems of design type consist in determining design characteristics of an engineering unit on the basis of given quality indices within certain limits. Required characteristics are causal with respect to these indices and limits.

In the case of the control, the role of causal characteristics is played by controlling influences the change in which creates the control action expressed by the system state, i.e. the effect.

It should be noted that there exists a fundamental difference between the two types of problems, between problems of diagnostics and identification and problems of design and control. In the case of design and control problems, the widening of the class of acceptable solutions usually simplifies things, since it is then necessary to find any practically feasible solution that would ensure the extremum of quality criterion with the given accuracy. At the same time, for identification and diagnostics problems the wider the class of possible solutions, the worse the situation. Specifically,

Third International Conference on Inverse Design Concepts and Optimization in Engineering Sciences
(ICIDES-III), Editor: G.S. Dulikravich, Washington D.C., October 23-25, 1991.

the errors of causal characteristics determined can increase which will make obligatory the use of regular methods of solution.

It should be noted that the theory and methodology of solution of inverse problems (that appear with diagnostics and identification of physical processes) are less developed than those for the other two classes of problems.

According to causal characteristics required it is possible to divide inverse problems of each group into various kinds. Most often, MMs of physical processes are based on equations with partial derivatives. In a general case, four kinds of inverse problems are introduced for them, viz., boundary, coefficient problem, retrospective problems, and geometric problems [1,2]. Boundary problems consist in finding functions and parameters that form boundary conditions; coefficient problems involve determining of functions and parameters that form part of equation coefficients; retrospective problems, (i.e. time reversed ones) consist in finding initial conditions; geometric ones presuppose reconstructing geometric characteristics of a domain or some points, lines or surfaces within a domain (for examples, determining co-ordinates of a phase transfer boundary or of a contact line of materials with different physical properties).

Now, if we again look at the block-scheme of development and creation of an important engineering object (see Fig. 1) we can point out possible and expedient fields of application of new methodology based on the solution of inverse problems. They are marked by shading. Thus, we can see that the scope of application of inverse problems to design and testing is rather wide. It can also be added that there exist a lot of useful applications of these methods for investigation, optimization and development of different technological processes as well.

INVERSE HEAT TRANSFER PROBLEM

Among the most developed and widely used in practice there are inverse problems of heat transfer. Consider now their posing.

In correspondence with three main forms of heat transfer let's introduce three groups of inverse problems: inverse problems of heat conduction, inverse problems of convective heat transfer, and inverse problems of radiative heat transfer. If combined or complex heat transfer is considered, corresponding statements of inverse problems will appear.

Let us now, for example, dwell upon a more concrete formulation of the two groups of inverse heat transfer problems.

INVERSE HEAT CONDUCTION PROBLEMS (IHCP). Problems of this kind are the best investigated and the most widely used in practice [1-4].

As an example, let us consider a one-dimensional problem of heat conduction in a two-layer plate assuming that the layer materials have different thermal properties and that in one of them there occurs a phase transfer, e.g., melting. Layer boundaries $b_1(\tau)$, $b_2(\tau)$, $b_3(\tau)$ can move with time as a result of some physical processes (ablation, thermal expansion or shrinking, mechanical deformation). The internal front of phase transfer $\eta(\tau)$ is also moveable.

We'll assume that temperature field $T(x, \tau)$ in the plate is described by equation system for generalized heat conduction

$$C_j \frac{\partial T_j}{\partial \tau} = \frac{\partial}{\partial x}(\lambda_j \frac{\partial T_j}{\partial x}) + K_j \frac{\partial Tj}{\partial x} + S_j, \quad j = 1, 2, 3 \tag{1}$$

in domains $b_1(\tau) < x < \eta(\tau)$, $\eta(\tau) < x < b_2(\tau)$, $b_2(\tau) < x < b_3(\tau)$, respectively. Conjunction conditions on lines $\eta(\tau)$ and $b_2(\tau)$ have the form

Third International Conference on Inverse Design Concepts and Optimization in Engineering Sciences (ICIDES-III), Editor: G.S. Dulikravich, Washington D.C., October 23-25, 1991.

$$T_1(\eta(\tau)-0,\tau) = T_2(\eta(\tau)+0,\tau)$$

$$\lambda_1 \frac{\partial T_1}{\partial x}\bigg|_{x=\eta(\tau)-0} - \lambda_2 \frac{\partial T_2}{\partial x}\bigg|_{x=\eta(\tau)+0} = r\frac{\partial \eta}{\partial \tau}$$

$$T_2(b_2(\tau)-0,\tau) = T_3(b_2(\tau)+0,\tau) - R\,\lambda_2 \frac{\partial T_2}{\partial x}\bigg|_{x=b_2(\tau)-0}$$

$$\lambda_2 \frac{\partial T_2}{\partial x}\bigg|_{x=b_2(\tau)-0} = \lambda_3 \frac{\partial T_3}{\partial x}\bigg|_{x=b_2(\tau)+0}$$

To the system (1) let us also add initial temperature distributions

$$T_j\,(x,0) = \xi_j\,(x), \quad j = 1, 2, 3$$

at $b_1(0) \le x \le \eta(0)$, $\eta(0) \le b_2(0)$, $b_2(0) \le x \le b_3(0)$, respectively, and conditions on the plate boundaries. As boundary conditions we can regard temperatures

$$T_j\,(b_j\,(\tau),\tau) = t_j\,(\tau), \quad j = 1, 3;$$

or heat fluxes

$$-\lambda_j \frac{\partial T_j}{\partial x}\bigg|_{x=b_j\,(\tau)} = q_j\,(\tau), \quad j = 1, 3;$$

or Newton conditions of convective heat transfer

$$-\lambda_j \frac{\partial T_j}{\partial x}\bigg|_{x=b_j\,(\tau)} = \alpha_j\,[T_j\,(b_j\,(\tau),\tau) - T_j^*\,(\tau)], \quad j = 1, 3;$$

or conditions that take into account body heat transfer with the environment by means of convection and radiation, and also the heat source that is caused by other processes (melting, sublimation, atom recombination, etc.)

$$-\lambda_j \frac{\partial T_j}{\partial x}\bigg|_{x=b_j\,(\tau)} = \alpha_j\,[T_j\,(b_j\,(\tau),\tau) - T_j^*\,(\tau)] + A_j\,q_{r_j} - \varepsilon_j\,\sigma\,T_j^4\,(b_j\,(\tau),\tau) + g_j, \quad j = 1, 3.$$

Here q_r is an incident radiant flux; σ - is the Stephan-Boltzmann constant. Various combinations of the above-mentioned boundary conditions on lines $b_1(\tau)$ and $b_3(\tau)$ are also possible.

Third International Conference on Inverse Design Concepts and Optimization in Engineering Sciences (ICIDES-III), Editor: G.S. Dulikravich, Washington D.C., October 23-25, 1991.

Coefficients C_j, λ_j, K_j and the source S_j in the equations in the general case can be functions of co-ordinate x, time τ, and temperature T_j, or any combination of these variables; in the simplest case they will be constant. Values $r, R, \alpha_j, A_j, \varepsilon_j, g_j$ can be considered as functions of the time and the corresponding temperature.

In the given problem, the causal characteristics will be volumetric heat capacities C_j, thermal conductivities λ_j, convection coefficients K_j, sources S_j, movement of boundaries b_1, b_2, b_3, and phase transfer front η; volumetric heat of phase transfer r, contact thermal resistance R, boundary temperatures t_j, heat fluxes q_j, ambient temperatures T_j^*; absorption coefficients A_j, emissivities ε_j; and surface heat sources g_j. The inverse problem of any kind consists in determining certain values of the sum total of causal characteristics adduced above. Certain additional conditions should be given. In most cases they will be temperature measurements $T(d_i, \tau) = f_i(\tau)$, $i = \overline{1, N}$ in N stationary or moving points d_i of a body; it is seldom that spatially continuous temperatures are considered.

According to the above-introduced causal characteristics of heat transfer processes, the following kinds of inverse problems can be introduced.

The first kind is a *retrospective heat conduction problem*, or the problem with reverse time - the finding of temperature distributions in previous moments (in other words - the determining of the prehistory of the given heat state);

The second kind is *a boundary inverse problem* - the reconstruction of thermal conditions at the boundary of the body. A problem connected with the continuation of the solution of heat conduction equation an overdetermined boundary belongs to this type of problems;

The third kind is *a coefficient inverse problem of heat conduction* - the specification of coefficient of the heat conduction equation (the identification of heat conduction operator).

Finally, it is possible to introduce one more kind of inverse problem, *a geometric* one that consists in finding some geometric characteristics of a heated body, e.g. in reconstructing the movement of the heat transfer boundary of a body on the basis of the results of temperature measurements within the body.

Combined statements are possible when causal characteristics of different types are sought simultaneously. For example, we can simultaneously estimate boundary conditions and temperature field in the past moments of time in the problem without initial conditions. This problem is a combination of a boundary problem and a retrospective one. There can exist natural combinations of a boundary problem and a coefficient one as well as those of a boundary problem and a geometric inverse problem of heat conduction.

INVERSE PROBLEM OF CONDUCTIVE-AND-CONVECTIVE HEAT TRANSFER FOR A POROUS BODY [2]. One more typical problem is connected with the development and testing of porous cooling systems of various designs. In these cases it is necessary to have information on the following characteristics: heat fluxes on blown surfaces; thermal conductivity λ_s; internal heat transfer coefficient α_V of a porous body, heat transfer coefficient α_0 at a coolant inlet into a porous material. Determination of these values from transient temperature measurements in porous structure is reduced to the solution of an inverse problem of conductive-and-convective heat transfer. In the one-dimensional case for a flat layer of a porous material with gaseous coolant, the MM of heat and mass transfer has the form

Third International Conference on Inverse Design Concepts and Optimization in Engineering Sciences
(ICIDES-III), Editor: G.S. Dulikravich, Washington D.C., October 23-25, 1991.

$$C_s \frac{\partial T_s}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda_s \frac{\partial T_s}{\partial x}\right) - \frac{\alpha_V}{1-P}(T_s - T_g),\ x \in (0, b),\ \tau \in (0, \tau_m]; \tag{2}$$

$$(\rho c_p)_g \frac{\partial T_g}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda_g \frac{\partial T_g}{\partial x}\right) - (\rho\, v\, c_p)_g \frac{\partial T_g}{\partial x} + \frac{\alpha_V}{P}(T_s - T_g), \tag{3}$$

$$x \in (0, b),\ \tau \in (0, \tau_m];$$

$$T_s(x, 0) = \xi_s(x),\ T_g(x, 0) = \xi_g(x), \tag{4}$$

$$-\lambda_s \frac{\partial T_s(b, \tau)}{\partial x} = \alpha_0[T_s(b, \tau) - T_{g0}]; \tag{5}$$

$$(\rho\, v\, c_p)_g T_g(b, \tau) = (\rho\, v\, c_p)_g T_{g0} + \alpha_0[T_s(b, \tau) - T_{g0}]; \tag{6}$$

$$-\lambda_s \frac{\partial T_s(0, \tau)}{\partial x} = q(\tau); \tag{7}$$

$$\frac{\partial^2 T_g(b, \tau)}{\partial x^2} = 0; \tag{8}$$

$$-\frac{dp_g}{dx} = \alpha(\mu v)_g + \beta(\rho v)_g^2; \tag{9}$$

$$\rho_g = \frac{P_g M_g}{8314\, T_g}. \tag{10}$$

Here indices s and g mean solid and gaseous phases respectively; c_p is specific heat capacity at constant pressure; ρ is density; v is velocity; capital P is porosity of the solid; a small letter p means pressure; μ is viscosity; M is molecular weight; α and β are hydraulic coefficients; T_{g0} is initial temperature of the injected gas.

This model contains the energy equations for solid and gaseous phases both the corresponding initial (4) and boundary conditions (5)-(8), and a modified Darcy's law (9) and equation of state for the gas (10). The condition (8) is one of the variants of natural boundary condition. It provides for the uniqueness of the direct-problem solution and, simultaneously, gives results that agree well with those corresponding to the actual boundary conditions of the first and second kind.

The unknown causal characteristics include $q, \lambda_s, \alpha_V, \alpha_0$.

The measurement data are specified with the conditions:

$$T_s(d_n, \tau) = f_n(\tau),\ \tau \in [0, \tau_m],\ n = \overline{1, N},\ N \geq 1,\ 0 \leq d_1 < d_2 <\ ` \ d_N \leq b$$

EXPERIMENTAL-THEORETICAL INVERSE-PROBLEM METHOD

In an exact formulation, any inverse problem can be written in compact form using an operator equation of the first kind

$$Au = f,\ \ u \in U,\ \ f \in F. \tag{11}$$

Here an operator A and right side f are given data. Value u is an unknown. It may be vector, function, or vector-function. Let us assume that operator A is continuous, and spaces U and F are metric.

It is known, that the problem (11) is called well-posed if it meets the following requirements (the Hadamard conditions):

* solution of the problem exists for any right side;

Third International Conference on Inverse Design Concepts and Optimization in Engineering Sciences
(ICIDES-III), Editor: G.S. Dulikravich, Washington D.C., October 23-25, 1991.

* solution is unique;
* it depends continuously on f.

If at least one of the requirements is violated, this problem is called ill-posed. This is the very situation, which is observed in solving the inverse problems.

This requires not only the development of special mathematical methods, but also proper technical organization of the studies. Experience indicated that only with a rational combination of physical and mathematical fundamentals it will be possible to make effective and creative use of the methods considered.

We shall use the concept of *an experimental-theoretical inverse problem method*, by which we mean an aggregate of studies and developments that includes physical and mathematical statement of the inverse problem, methods and algorithms for its solution, the necessary technical systems, and organization of experimental studies.

ON THE HISTORY OF THE MATHEMATICAL SUBJECT-MATTER

A retrospective look at the matter of solving inverse heat transfer problems and utilization of corresponding methods justifies to the fact that a tendency for rapid development of the scientific trend observed to-day was of irregular nature before.

The interest and attention shown by investigators to this problem appeared incidentally. The first formulations and first attempts of solving inverse problems, perhaps, should be related to determination of historical climate and heat condition of earth's ground layer. These are works of Fourier, Poisson and Kelvin in the 19th century.

It should be noted that some methods used at present are based on solutions known long enough. The example of this - presentation of solutions of linear problems of heat conduction through Dugamel integral (1832) with further numerical inversion of it. However, the corresponding procedures for determining unsteady heat fluxes appeared much later in works by T.J. Mirsepassi, one of the first having been published in 1958 [5], in works by G. Stolz (1960) - [6], by J.V. Beck (1962 and later in [7,8]), by G.T. Aldoshin, A.S. Golosov, V.I. Zhuck (1968 and later in [9,10]) by O.M. Alifanov (1969 and later in [11-14]) and by other authors. Regularization of heat state of solid bodies in the form of exponential law of temperature change was discovered in 1901 by J. Boussinesq. At the same time the basics for the theory of regular heat state was developed by G.M. Kondratiev and later by A.V. Lyikov in the 40s and the 50s. In 1955 the principle of regular heat state was used by N.V. Shumakov to find non-stationary heat fluxes through a successive interval method [15]. Apparently, it is the first "promulgated" technique for solving boundary inverse problems of heat condition.

Note that for a particular case of so-called pseudo-inverse heat condition problem W.H. Giedt in 1955 [16] and O.N. Kastelin jointly with L.N. Bronsky in 1956 [17] published a procedure for its solution which still finds its application.

A solution of heat conduction problem in the Cauchy generalized formulation presented as an infinite power series was obtained by J. Stefan in 1890 [18]. This result can be considered as the first exact solution of a one-dimensional inverse problem with constant coefficients, although for this purpose it was not used until the studies of A.G. Tyomkin and O.R. Burggraf [19, 20] who in 1961 and 1964, respectively, got similar by form solutions for a series of other linear inverse problems of heat conduction.

Thus, despite the fact that necessary preconditions for constructing solution of inverse problems appeared already in the last century and at the very beginning of the current century, practical

Third International Conference on Inverse Design Concepts and Optimization in Engineering Sciences
(ICIDES-III). Editor: G.S. Dulikravich, Washington D.C., October 23-25, 1991.

conclusions, nevertheless, have been drawn quite recently. The most active and stable period for the development of solution methods and their application falls on the last 20 years.

Let us touch upon history of mathematical studying and solving ill-posed problems.The conditions for well-posed formulation of any problem of mathematical physics were introduced by J.Hadamard in 1902 [21]. Usually it was assumed that if the original mathematical formulation of a problem did not satisfy any of these conditions, it was then of no physical or practical sense, and, consequently, there was no reason of constructing its solution. Gradually, however, the attitude of mathematicians and physicists towards ill-posed problems began to change. Already in 1926 T. Carleman makes the first attempt to solve an ill-posed problem [22]. In the 3Os new investigations on determination of historical climate have been made by A.N. Tikhonov. In 1943 he formulated for the first time in a complete form the so-called *conditionally-ill-posed statement* of ill-posed problem of mathematical physics assuming a stable solution in the compact class of functions [23]. This fundamental result, beginning from 1953, is further developed in the works by M.M. Lavrentiev and by V.K. Ivanov (see bibliography in [24, 25]). To this trend we can refer an interesting study by F.John in which he presents a method of solving heat conduction equation with inverse time [26].

The most weightful mathematical result of general nature in the area of ill-posed problem opening a fruitful direction in the mathematical physics and computing mathematics was obtained in 1963 by A.N. Tikhonov [27]. It should be noted that very close idea was proposed for solving linear integral equations of the first kind by Phillips in 1962 [28]. But he did not give any strict substantiation of this approach. Tikhonov's method of *regularization* broadened considerably the bounds of effective practical use of ill-posed problems in various fields of science and technology. Since that time this method has got intensive development in the works by A.N.Tikhonov, V.K. Ivanov, V.Ya. Arsenin, V.A. Morozov, A.B. Bakushinsky, V.B. Glasko, V.N. Strakhov and many other mathematicians (see bibliography in [25, 29]).

At present we have quite a complete mathematical theory of solving ill-posed problems, the pivot of which being this very method.

The majority of works devoted to a development of the regularization method treat one of its forms which got the name of a *variational method.*

Other forms are also possible. Among the most universal is a so called *iterative regularization* which is most effectively realized with the help of non-linear gradient algorithms. This quite a general method has been proposed by O.M.Alifanov [30, 31] and mathematically grounded together with S.V.Rumyantsev [32, 33]. Important contribution to solving inverse heat conduction problems by the iterative regularization has been made by E.A. Artyukhin.

Also, it is necessary to mention a book of R. Latt'es and J.-L. Lions [34] in which they suggest the quasi-inversion method specially for the equations with partial derivatives. A close approach was suggested by O.M. Alifanov in 1971 for solving inverse heat conduction problem in the Cauchy statement [13]. It is called the *artifical hyperbolization* method. But these approaches haven't strict substantiation.

Simultaneously with the development of the general theory of ill-posed problems and construction of regular method for their solution a process is observed with respect to the elaboration of stable and effective in practice methods and algorithms for solving inverse problems of heat conduction. The initial phase of developing the computational procedures to solve these problems (till the time when a regularization method appeared in 1963 and, evidently, after another few years when the attention of practical workers was attracted by this method, i.e. somewhere in 1968-1970)

Third International Conference on Inverse Design Concepts and Optimization in Engineering Sciences (ICIDES-III), Editor: G.S. Dulikravich, Washington D.C., October 23-25, 1991.

can be named *a heuristic regularization* and the corresponding methods got a conventional term of *direct methods*. In other words the authors of corresponding algorithms achieved stability and acceptable accuracy of results basing mainly on the physical sense and, consequently, on the physical level of rigour. Apart from the above works to this trend in solution of inverse problems we can refer *a trial-and-error method* used by L.A. Kozdoba [35] and methods of *linear dynamic filtration* being developed by Yu.M. Matsevitiy, A.V. Multanovsky and D.F. Symbirsky [36, 37]. Rigorous mathematical conditions are not yet formulated in the approaches pointed above.

Alongside with heuristic methods, beginning from the end of the 60s and in early the 70s, there appeared mathematically rigorous methods of solving inverse heat conduction problems.

In their majority these methods are related to the linear problem formulation and constructed basing on a variational technique of regularization and, later on, on iterative regularization. Just to illustrate this, refer to some works both on the first [1, 38-41] and on the second [2, 4, 30, 42] directions. Both approaches, as computational experiments and actual physical tests show, turn out to be acceptable for solving various nonlinear problems as well [1, 2-4, 31, 43-46].

APPLICATIONS OF INVERSE-PROBLEM METHODS

Numerous scientific and practical results have now been obtained with the aid of the pertinent methods. Let us briefly dwell on some of them.

HEAT DIAGNOSTICS. Let us start with non-stationary heat diagnostics [2,41,47]. The method of boundary inverse heat conduction problems can be used in thermal diagnostics of both slow and fast heat transfer processes. Our investigations have demonstrated that it is possible to reconstruct heat-flux and heat transfer coefficients with accuracy comparable to that of temperature measurements in the solid body. We have developed different principles of one-, two- and three-dimensional thermal indirect measurements based on solution of boundary inverse problems, which have required dimensionality.

On the basis of these principles, sensing devices for heat diagnostics of high-temperature gas flows has now been designed, refined experimentally, and put to practical use in various branches of industry. In particular, these are different types of uncooled and cooled sensors. For example, similar sensors are used for experimental studies on plasmatrons and gasdynamic stands in which the gas jets are created by special aviation and rocket engines.

Similar sensors can be used successfully to measure not only convective, but also radiative heat fluxes. They are capable of much faster response rates than the Gardon-type sensors widely used in practice.

Experimental studies showed that heat-flux variations at frequencies up to 100 Hz can be registered by using uncooled sensors and processing their readings by solving a boundary IHCP.

One-dimensional sensors can be used to measure transient local heat fluxes and local heat transfer coefficients. To determine discrete fields of these values it is necessary to install a sufficient number of sensors at various space points, for example, at various points of streamlined surface of a solid body. However, if we go to solution of two-and three-dimensional inverse problems of heat conduction, we can reconstruct continuous spatial-time dependences of heat fluxes and heat transfer coefficients on a body surface. In these cases temperature measurements are usually made on part of a heat-insulated boundary of the body, namely on a line for a two-dimensional case and on a surface for a three-dimensional case. Sensors with such sensitive elements [2,47] can be mounted on a model or a mock-up of the object under study, or on a full-scale object, the thermal conditions

Third International Conference on Inverse Design Concepts and Optimization in Engineering Sciences (ICIDES-III), Editor: G.S. Dulikravich, Washington D.C., October 23-25, 1991.

of which is determined under test or design operating conditions. Sometimes temperature measurements can be conducted within a solid body.

The above methods for indirect measurements are of special value in the diagnostics of heat-transfer processes under various conditions that do not admit of easy calculation, as in investigation of the laminar to turbulent flow transition, the interaction of shock waves with boundary layers, heat transfer in separation zones, streamlining by nonequilibrium flows of dissociated gas, in the case of heat exchange with boiling, injection of gas or liquids into boundary layers, and so forth.

It is important to note, that the procedures of *simultaneous determination* in experiment of the two or more functions (or parameters) in heat balance equation on body surface are developed [2,47]. For example, we can find simultaneously a local coefficient of convective heat transfer as a function of temperature factor α (Tw/T*) and an emissivity of the surface as a function of its temperature ε (Tw) for known environment characteristic temperature $T^*(\tau)$. Basis of these procedures is special formulations and solution methods of boundary IHCPs.

The boundary inverse-problem method is one of basic for study of *non-stationary heat transfer* in the system: solid-gas (or liquid).

It is known, that a heat transfer coefficient, obtained for conditions when an influence of solid body on thermal state of boundary layer is taken into account can considerably differ from a heat transfer coefficient, which is determined for stationary conditions. The approach to study non-stationary heat exchange includes two parts. The first one consists in solution of joint heat transfer problems, when equations of heat-and-mass transfer both for solid and gas (or liquid) must be solved simultaneously.

The second is experimental investigations of non-stationary heat transfer and, in many cases, the experiment still remains the major technique of such studies [48,51]. Such experimental investigations are based on simulation of natural transient heat-and-mass transfer and determination of non-stationary heat transfer coefficients as functions of time. It is required not only to correctly conduct and successfully carry out experimental research, but also (and this is very important) to find effective ways of processing the obtained data. It was found that inverse problem forms an effective means of getting the necessary results in experimental information processing.

Use of the inverse-problem methods to process experimental data permits to develop *new approaches to the very formulation of the experiments* to investigate heat and mass transfer, making such experiments more efficient and informative. For example, a new universal procedure has been proposed for aerodynamic thermal tests to investigate heat transfer in a broad range of Reynolds numbers using working chambers of comparatively small sizes [52]. This technique is based on the use of the boundary inverse heat conduction problem, that has made it possible to conduct experiments under essentially nonsteady heat-transfer conditions with long models mounted in the working section of the wind tunnel before it is started (which had previously been impossible). Part of the model is situated directly in the supersonic nozzle. This makes it possible to investigate flows with uniform fields of the gasdynamic parameters over practically the entire characteristic rhombus, and this, in its turn, makes it possible to set up not only laminar but also transitional and turbulent boundary layers on the model.

Another area of application of those methods relates to investigation of *temperature fields, heat flux-fields and also thermal stresses* in structural materials, something that is very important for various types of flight vehicles, engines, and power-generating equipment [47]. It is often found that temperature sensors cannot be mounted inside of materials due to technological, structural and

Third International Conference on Inverse Design Concepts and Optimization in Engineering Sciences
(ICIDES-III). Editor: G.S. Dulikravich, Washington D.C., October 23-25, 1991.

methodical reasons (because of its violating the integrity, and strength properties of materials, introducing distortions into the temperature field and into the field of thermal stresses, and also due to the difficulties in providing good thermal contacts of sensors with the material, etc). It is then necessary to reconstruct the temperature field from temperature and heat-flux measurements made on part of the boundary of the body, i.e. to solve the corresponding inverse problem. This approach has been used, for example, in investigating the hot strength of graphite structures, and has produced good results.

New and important field of use of methods based on solution of the inverse problems is experimental-theoretical studies of heat-and-mass transfer in porous mediums, in particular, *porous cooling systems*. These systems are an effective means of heat protection. It is performed by the coolant supply through special inserts made of porous materials. Coolant here is gas or liquid. In the course of experimental studies of porous cooling systems it is necessary to determine non-stationary thermal boundary conditions on the surface of a porous body and to identify heat effect of coolant injection into a boundary layer. The direct measuring of values included into the boundary conditions of a heated surface is either very difficult or downright impossible, but the temperature on the opposite surface of a solid matrix can be measured. In this case we are faced with the necessity of solving a boundary inverse problem for an equation system for heat-and-mass transfer in a porous structure [2,53,55]. For a gaseous coolant appropriate formulation of inverse problem was considered above.

Of practical importance is the problem of studying the *heating and heat destruction of thermal protective materials*, including the investigation of reducing convective heat transfer due to injection of gaseous products from the ablated surface. The main types of measurements in experimental study of such materials are temperature measurements within the bodies (usually by means of thermocouples) and on the external surface (by optical methods) and measurements of the ablation rate. The processing of measurement data can be performed by methods based on the solution of inverse heat conduction problems.

The following example is referred to a determination of *thermal properties of different medium and materials*, in particular, heat-protective materials interacting with high-enthalpy gas flow. Thermophysical measurements, based on classical techniques, for many materials can be made only at temperatures and rates of heating much less than those in reality. To avoid the above discrepancy is possible simulating the required conditions of specimen heating on special test facilities (plasmatrons, in the jets of rocket engines and other) with a successive treatment of temperature measurements by coefficient inverse problem [56-64]. That is, using some mathematical model of heat transfer in the material (in the simplest case - a heat conduction equation) we are to find a required value (or values), for example a heat conductivity as function of temperature, "adjusting" the calculated temperatures to those thus measured. Thermal properties thus obtained correspond to the heating conditions brought near to real conditions in which the material operates. In many cases, if properties of decomposing materials are investigated, it is necessary to develop inverse problem procedure for mathematical model that takes account of the non-isothermal decomposition kinetics.

Another field is *the estimation of contact resistances* which characterize the heat transfer between the connected parts of structures as well as the prediction of their change in the course of time, in particular for structures, where there is a great number of bolted and riveted joints, hinges and so on. For thermal shields it is necessary to know *the resistance of adhesive film*, and this problem often can be interpreted as the problem of contact resistance specification. The method of

Third International Conference on Inverse Design Concepts and Optimization in Engineering Sciences
(ICIDES-III). Editor: G.S. Dulikravich, Washington D.C., October 23-25, 1991.

boundary inverse problem can be successfully applied to processing the results of specially conducted experiments in solving the problem of contact heat transfer, non-stationary conditions included.

The next field of application of inverse problems is *diagnostics of friction*. In mechanical engineering, the investigations of friction and wear of different movable joints are of great importance because machines reliability and overhaul period depend on them. Besides, these investigations permit to reduce friction losses and, consequently, increase machines efficiency. Today, bench tests is very often the only means to test experimentally a movable joint. But they can not substitute service tests which provide the most complete data on a joint performance in operating conditions. At the same time, service tests of friction units rarely give data on friction losses. Thus, for example, the existing methods of direct measuring of friction torque, characterizing work in friction, rest on the use of special elastic elements, i.e. torsion devices. Their lay out presents a problem even in bench tests. In operating conditions measuring of friction torque with these devices is often impossible. So, work in friction (friction torque) is defined through other measurements well correlated with the sought-for quantity. The most suitable are temperature measurements not requiring complex equipment. Using these data it is possible to reconstruct heat release in friction zone. Almost all friction energy (85-100%) goes into heat. Thus, it becomes possible to estimate work in friction, and, accordingly, friction torque, using the data on heat release. Heat release itself may be found by solving inverse heat transfer problem with known temperature measurements.

Using this approach and iterative regularization, appropriate procedure for tests of the sliding bearing was developed and used in practice [65]. Obtained results of 10-15% agree with the results of torsion measurements.

The above applications concern *diagnostic and identification problems*. General procedure for structural and parametric identification of physical processes, based on solving ill-posed inverse problems, is presented in [66].

With the help of inverse problem principle various *design problems* also may be solved. The problem of the optimal design of a multilayes heat shield is considered. It is required to determine the design characteristics (the number, materials and thicknesses of the layers) of such shield, one of whose boundaries as well as the corresponding layer is subjected to external transient heating and ablation, while the other is subjected to cooling by the circulating heat transfer agent. The total mass of the shield is the criterion for the quality of the heat protection. The optimization problem has a number of restrictions taken into account, which are dictated by the requirements of the admissible temperature conditions for the layers, the specific heat of the coolant, the thermal stresses, and so on. Therefore we have a combined coefficient - geometric inverse problem of thermal design type. This problem is solved by the iterative method [67,68].

At present methods based on solution of different inverse problems find their application not only for model thermal experiments and parameter optimization. They are also used for *full-scale tests*, in particular, for diagnostics of heat transfer boundary conditions and heat loads on different real structures and for identification of thermal properties of heat protection and heat insulation materials in real operating conditions.

One field of application of these methods is *thermal-vacuum test of spacecrafts*. Such approach permitted to create new effective procedure of testing. It includes the following three main parts: special preliminary testing of object for the purpose of identification and correction of mathematical

Third International Conference on Inverse Design Concepts and Optimization in Engineering Sciences (ICIDES-III). Editor: G.S. Dulikravich, Washington D.C., October 23-25, 1991.

models of heat transfer processes in test object; choice of thermal simulator mode with help of solution of the inverse problem of control type; regular testing itself.

Very important fields of applications of inverse problems are different *nature (real) experiments and tests*, for example, flight tests. In many of these cases, such approach is the only possible means for obtaining necessary quantitative information about heat conditions of vehicles under test, since other methods turn out to be unfit. Appropriate procedures and technical devices were created and used for study (in flight experimental conditions) of porous cooling system, reusable thermal protection, thermocontrol coating of space vehicles and strength of structures of flight vehicles.

For example, developed methods were applied to study of *thermal modes of reusable heat protection* of "Buran" aerospace vehicle. The flight tests were conducted on special automatic re-entry vehicles "Bor-4" series. In these cases heat diagnostics was carried out in the following ways:

- estimation of heat fluxes on the surface of the tiled thermal shield;
- quality analysis of the effects of physical-chemical reactions on the thermal shield surface with its catalytic properties being changed;
- evaluation of heat state of the thermal shield surface in the tiles gaps;
- estimation of the inner heat state of the tiled thermal shield material under the heating in flight conditions.

Unique results were obtained by means of these methods in the course of such tests.

The next important example has to do with diagnostics of *radiative characteristics of thermal control coatings* of spacecrafts. Of great interest is an experimental determination of the solar radiation integral absorption factor and integral semi-spherical emissivity of external surface in the conditions of actual operation of the coatings. In particular, such studies are conducted on vehicles of "Cosmos", "Meteor", "Meteor-Priroda" series. In the result it was possible to construct a mathematical model for varying the radiation characteristics of coatings in the course of time and predict these variations for longer time of operational use of the vehicle, as compared with duration of experiment.

Besides model experiments, design and testing of technical units inverse problems find their fruitful use in investigations, optimization and operating diagnostics of *various technological processes*. Just for example let us touch upon some of them.

Procedures for determination of heat loads by inverse-problem solution may be very helpful in experimental study of *liquid cooling* in continuous casting and heat-treatment of metals. Such cooling removes heat flows of rather high specific rate - up to 100 mWt/m^2 with realization of high velocity non-stationary processes. Complex thermohydrodynamic processes occurring while spraying liquid over a high temperature surface cannot be described so far with the required accuracy by means of theoretical methods. So, such kind of investigations are still described through experiments. The experimental data are obtained and generalized by solving inverse problems of heat exchange.

The direct estimation of local rates of the removed heat flow during liquid cooling and with boiling is hindered by rather great change of surface temperature rate. Standard heat flux meters have time constant about 1 sec - two orders more than process characteristic time. Effective measuring means for these purposes may be obtained on the basis of boundary inverse problems principles [70].

Another example is the thermofretting of metals. This is a progressive trend in heat-treatment technology for critical steel products that operate under heavy mechanical loads, such as the disks and rotors of large power-generating steam turbines. It is now impossible to investigate and optimize the thermofretting process without experimental testing, and this is a typical area in which inverse-problem methods can be used to good effect.

Inverse problems of structure mechanics. Problems of reconstruction of loads on the structure by its stress-deformed state parameters as well as problems of determination of the fields of stresses and shears in a given part of construction elements by stresses (shears) values on a part of its surface fall under the class of inverse problems of mechanics of deformated solid [69]. An analysis is made of corresponding methods and their practical application for investigation of strength of space vehicles during flight tests.

Of course, the range of possible practical uses of the inverse-problem methods is considerably broader than that indicated above.

To summarize, we observe that these experimental-theoretical methods not only have a broad spectrum of important applications, but they are distinguished of high information yield and enough high reliability. For more complete acquaintance with existing today methods and algorithms of solving ill-posed inverse problems and their different applications refer to the following books [1-4,25,29,34-37,41,48]. Also, it can be recommended to look through the numbers of Inzh.Fiz.Zh: vol.29, no.1, 1975; vol.33, no.6, 1977; vol.39, no.2, 1980; vol.45, no.5, 1983; vol.49, no.6, 1985; vol.56, no.3, 1989 (English translation in Journal of Engineering Physics - bibliography data is the same). The numbers were dedicated specially to those problems.

REFERENCES

1. O.M.Alifanov. Identification of Heat Transfer Processes of Flight Vehicles. - Moscow: Mashinostroenie, 1979, 216 p. (in Russian).

2. O.M.Alifanov. Inverse Heat Transfer Problems. - Moscow: Mashinostroenie, 1988, 280 p. (in Russian).

3. J.V.Beck, B.Blackwell, C.R.St.Clair, Jr. Inverse Heat Conduction. Ill-posed Problems. - A Wiley-Interscience Publication, New Jork, 1985, 308 p.

4. O.M.Alifanov, E.A.Artyukhin, S.V.Rumyantsev. Extreme Methods of Solving Ill-posed Problems and Their Application to Inverse Heat Transfer Problems. - Moscow: Nauka, 1988, 288 p. (in Russian).

5. T.J.Mirsepassi. Heat Transfer Charts for Time-Variable Boundary Condition. Part I - Semi-Infinite Solid. - ASME Paper 58-SA-7, 1958.

6. G.Stolz. Numerical Solutions to an Inverse Problem of Heat Conduction for Simple Shapes. - J.Heat Transfer, 1960, v.82, No. 1, pp.20-26.

7. J.V.Beck. Calculation of Surface Heat Flux from an Internal Temperature History. - ASME Paper 62-HT-46, 1962, 11 p.

8. J.V.Beck. Surface Heat Flux Determination Using an Integral Method. - Nucl.Eng.Des., 1968, v.7, No.2, pp.170-178.

9. G.T.Aldoshin, A.S.Golosov, V.I.Zhuk. Solution of an inverse problem of nonstationary heat conduction for a slab. Heat and mass transfer. Minsk, Naura i Teknika. v.8, 1968, pp.186-199 (in Russian).

Third International Conference on Inverse Design Concepts and Optimization in Engineering Sciences (ICIDES-III). Editor: G.S. Dulikravich, Washington D.C., October 23-25, 1991.

10. V.I.Zhuk, A.S.Golosov. Inzh.Fiz.Zh., 1975, vol.29, No. 1, pp.45-50 (Engl. transl. in: J.Eng.Phys., 1975, vol.29, No.1).

11. O.M.Alifanov. Solution of a Non-Stationary Heat Conduction Problem for Investigation of Ablation of Heat Protection Materials. - In: Preprints to the All-Union Conference of Young Scientists on Heat-and-Mass Transfer (January 27-30, 1969), Minsk: 1969, 187 p. (in Russian).

12. O.M.Alifanov. Solution of the Problem of Nonstationary Heat Conduction and its Application for Studying the Heat Shield Materials. - In: Investigations of Nonstationary Convective Heat and Mass Transfer. - Minsk: Nauka i Tekhnika, 1971, pp.322-333 (in Russian).

13. O.M.Alifanov. Study of Direct and Inverse Heat Conduction Problems Conformably to Design of Thermal Protection of Flight Vehicles. Doctorate thesis, Moscow: Moscow Aviation Institute, 1971 (in Russian).

14. O.M.Alifanov. Inzh.Fiz.Zh., 1973, vol.24, No.6, pp.1129-1137. (Engl.transl. in: J.Eng.Phys., 1973, v.24, No.6).

15. N.V.Shumakov. Methods of Experimental Study of Solid Body Heating. Doctorate thesis, Moscow: Energy Institute named after G.M.Krzhizhanovsky, 1955 (in Russian).

16. W.H.Giedt. The Determination of Transient Temperatures and Heat Transfer at a Gas-Metal Interface Applied to a 40 mm Gun Burrel. - Jet Propulsion, 1955, v.25, No.4, pp.158-162.

17. O.N.Kastelin, L.N.Bronsky. Heating of a Blunt Body in the Supersonic Flow. - In: Physical Gasdynamics, Heat Transfer and High Temperature Gas Thermodynamics. Moscow: Publ.H. of USSR Acad. of Sciences, 1962, pp.233-238 (in Russian).

18. J.Stefan. On the Theory of Ice Formation, Especially on Ice Formation in Polar Seas (Uber die Theorie der Eisbildung, insbesondere uber die Eisbildung in Polarmeere). Sitzungsberichte der Kaiserlichen Akademic Wiss, Wien., Math. naturwiss. Kl., 1890, v.98(2a), pp.965-973 (in German).

19. A.G.Tyomkin. Determination of Variable Outside Heat Effects by Methods of Heat Conduction. - Izv. Vyssh. Uchebn. Zaved., Energetika, 1961, No. 5, pp.60-71 (in Russian).

20. O.R.Burggraf. An Exact Solution of the Inverse Problem in Heat Conduction Theory and Applications. - J.Heat Transfer, 1964, v.86c., p.373-382.

21. J.Hadamard. Lectures on Cauchy's Problem in Linear Partial Differential Equations. - Yale University Press, New Haven, CT, 1923.

22. T.Carleman. Les fonctions quasi analytiques. Paris (1926),

23. A.N.Tikhonov - Dokl. Akad. Nauk SSSR, 1943, v.39, No.5, pp.195-198 (Engl. transl. in: Soviet Math. Dokl., 1943, v.39, No.5).

24. M.M.Lavrentiev. On Some Ill-posed Problems of Mathematical Physics. - Novosibirsk: Izd. Sib. Otd. Akad. Nauk SSSR, 1962, 92 p. (in Russian).

25. A.N.Tikhonov, V.Ya.Arsenin. Methods of Solving Ill-Posed Problems. - Moscow: Nauka, 1986, 288 p. (in Russian).

26. F.John. Numerical Solution of the Heat Equation for Preceding Times. Ann. mat. pura ed. appl., 4, 40(1955), pp.129-142.

27. A.N.Tikhonov - Dokl. Akad. Nauk SSSR, 1963, v.151, No.3, pp.501-504 (Engl. transl. in: Soviet Math. Dokl., 1963, vol.151, No.3).

28. D.L.Phillips. A Technique for the Numerical Solution of Certain Integral Equations of the First Kind. - J.Assoc. Comput. Mach. 1962, v.9, No.1, pp.84-97.

29. V.A.Morozov. Regular Methods of Solving Ill-posed Problems. - Moscow: Izd. Mosc. State Univ., 1974, 359 p. (in Russian).

30. O.M.Alifanov. Inzh. Fiz. Zh., 1974, v.26, No.4, pp.682-689 (Engl. transl. in: J.Eng. Phys., 1974, v.26, No.4).

31. O.M.Alifanov. Teplofiz. Vys. Temp., 1977, v.15, No.3, pp.598-605 (Engl. transl. in: High Temperatures, 1977, v.15, No.3).

32. O.M.Alifanov, S.V.Rumyantsev. Dokl. Akad. Nauk SSSR, 1979, v.248, No.6, pp.1289-1291 (Engl. transl. in: Soviet Math. Dokl., 1979, v.248, No.6).

33. O.M.Alifanov, S.V.Rumyantsev. Inzh. Fiz. Zh., 1980, v.39, No.2, pp.253-258 (Engl. transl. in: J.Eng. Phys., 1980, v.39, No.2).

34. R.Lattes, J.-L. Lions. Methode de Qusi-Reversibilite et Applications. Dunod. Paris. 1967.

35. L.A.Kozdoba, P.G.Krukovsky. Methods of Solving Inverse Problems of Heat Transfer. - Kiev: Naukova Dumka, 1982, 359 p. (in Russian).

36. Yu.M.Matsevity, A.V.Multanovsky. Identification in the Problems of Heat Conduction. - Kiev: Naukova Dumka, 1982, 240 p. (in Russian).

37. D.F.Simbirsky. The temperature diagnostics of engines. Kiev: Tekhnika, 1976, 208 p. (in Russian).

38. A.N.Tikhonov, V.B.Glasko. - Zh. Vychisl. Mat. Mat. Phys., 1967, v.7, No.4, pp.910-914 (Engl. transl. in: USSR Comput. Math. and Math. Phys., 1967, v.7, No.4).

39. O.M.Alifanov. Regularization of Solutions of Inverse Heat Conduction Problems. - In: Heat and Mass Transfer. - Minsk: Nauka i Tekhnika, v.8, 1972, pp.89-98 (in Russian).

40. O.M.Alifanov, Inzh. Fiz. Zh., 1973, v.24, No.2, pp.324-333 (Engl. transl. in: J.Eng. Phys., 1973, v.24, No.2).

41. O.M.Alifanov, V.K.Zantsev, B.M.Pankratov, et al. Algorighms for Diagnostics of Heat Loads of Flight Vehicles. Under the editorship of Academician V.P.Mishin. - Moscow: Mashinostroenie, 1983, 168 p. (in Russian).

42. E.A.Artyukhin, S.V.Rumyantsev. Inzh. Fiz. J., 1980, v.39, No.2, pp.264-269 (Engl. transl. in: J. Eng. Phys., 1980, v.39, No.2).

43. E.A.Artyukhin. Inzh. Fiz. J., 1981, v.41, No.4, pp.587-592 (Engl. transl. in: J. Eng. Phys., 1981, v.41, No.4).

44. O.M.Alifanov, E.A.Artyukhin. Teplofiz. Vys. Temp., 1978, v.16, No.4, pp.819-825 (Engl. transl. in: High Temperatures, 1978, v.16, No.4).

45. O.M.Alifanov, V.V.Mikhailov. Inzh. Fiz. Zh., 1978, v.35, No.6, pp.1124-1129 (Engl. transl. in: J.Eng. Phys., 1978, v.35, No.6).

46. O.M.Alifanov, I.E.Balashova. Izv. Sib. Otd. Akad. Nauk SSSR, Tekhn. nauki, 1987, No.2, pp. 5-11 (in Russian).

47. V.P.Mishin, O.M.Alifanov. Mashinovedenie. Part 1: 1986, No.5, pp.19-29 (Engl. transl. in: Soviet Mashine Science, 1986, No.5, pp.16-25); Part 2: 1986, No.6, pp.11-21 (Engl. transl. in: Soviet Mashine Science, 1986, No.6, pp.8-16).

48. N.V.Shumakov. A Method of Successive Intervals in Thermometry of Transient Processes. - Moscow: Atomizdat, 1979, 216 p. (in Russian).

49. E.K.Kalinin, I.I.Berlin, V.V.Kostyuk, et al. Inzh. Fiz. Zh., 1972, v.22, No. 4, pp.610-613 (Engl. transl. in: J.Eng. Phys., 1972, v.22, No.4).

50. O.M.Alifanov, M.I.Gorshkov, V.K.Zantsev, B.M.Pankratov. Inzh. Fiz. Zh., 1975, v.29, No.1, pp.26-30 (Engl. transl. in: J.Eng. Phys., 1975, v.29, No.1).

Third International Conference on Inverse Design Concepts and Optimization in Engineering Sciences
(ICIDES-III). Editor: G.S. Dulikravich, Washington D.C., October 23-25, 1991.

51. V.L.Sergeyev, L.A.Sergeyeva. Study of Non-stationary Heat Transfer for Flat and Axially Symmetric Flows. Preprint No.5 - Minsk: Institute of Heat-and-Mass Transfer, 1985, 63 p. (in Russian).

52. O.M.Alifanov, N.T.Batura, A.M.Bespalov, et. al. Inzh. Fiz. Zh., 1977, v.33, No.6, pp.988-992 (Engl. transl. in: J. Eng. Phys., 1977, v.33, No.6).

53. O.M.Alifanov, E.A.Artyukhin, A.P.Tryanin. Teplofiz. Vys. Temp., 1983, v.21, No.6, pp.1160-1168 (Engl. transl. in: High Temperatures, 1983, v.21, No.6).

54. O.M.Alifanov, A.P.Tryanin. Inzh. Fiz. Zh., 1985, v.48, No.3, pp.472-483 (Engl. transl. in: J. Eng. Phys., 1985, v.48, No.3).

55. O.M.Alifanov, A.P.Tryanin, A.Lozhkin. Inzh. Fiz. Zh., 1987, v.52, No.3, pp.461-468 (Engl. transl. in: J. Eng. Phys., 1987, v.52, No.3).

56. Yu.V.Polezhaev, F.B.Yurevich. Heat Protection. Moscow: Energiya, 1976, 392 p. (in Russian).

57. K.G.Omelchenko, M.V.Savelov, V.P.Timoshenko. Teplofiz. Vys. Temp., 1974, v.12, No.4, pp.761-768 (Engl. transl. in: High Temperatures, 1974, v.12, No.4).

58. V.M.Yudin. Distribution of heat in glass-reinforced plastic. - Proceedings of TsAGI, No. 1267, - Moscow: Izd. TsAGI, 1970, 41 p. (in Russian).

59. O.M.Alifanov, M.V.Klibanov. Inzh. Fiz. Zh., 1985, v.48, No. 6, pp.998-1003 (Engl. transl. in: J.Eng. Phys., 1985, v.48, No.6).

60. E.A.Artyukhin. Teplofiz. visok. temp., 1981, v.19, No.5, pp.963-967 (Engl. transl. in: High Temperatures, 1981, v.19, No.5).

61. E.A.Artyukhin, V.E.Kullikh, A.S.Okhapkin. Inzh. Fiz. Zh., 1983, v.45, No.5, pp.788-794 (engl. transl. in: J. Eng. Phys., 1983, v.45, No.5).

62. A.S.Okhapkin. Inzh, Fiz. Zh., 1985, v.49, No.6, pp.989-994 (Engl. transl. in: J. Eng. Phys., 1985, v.49, No.6).

63. A.A.Goryachev, V.M.Yudin. Inzh. Fiz. Zh., 1982, v.43, No.4, pp.641-648 (Engl. transl. in: J. Eng. Phys., 1982, v.43, No.4).

64. P.V.Prosuntsov, S.V.Reznik, Inzh. Fiz. Zh., 1985, v.49, No.6, pp.977-982 (Engl. transl. in: J. Eng. Phys., 1985, v.49, No.6).

65. I.N.Chersky, O.B.Bogatin, N.P.Starostin. Inzh. Fiz. Zh., 1984, v.47, No.6, pp.1000-1006 (engl. transl. in: J. Eng. Phys., 1984, v.47, No.6).

66. O.M.Alifanov, Inzh. Fiz. Zh., 1985, v.49, No.6, pp.889-897 (Engl. transl. in: J. Eng. Phys., 1985, v.49, No.6).

67. O.M.Alifanov. Inzh. Fiz. Zh., 1977, v.33, No.6, pp.972-981 (Engl. transl. in: J. Eng. Phys., 1977, v.33, No.6).

68. V.V.Mikhailov. Inzh. Fiz. Zh., 1980, v.39, No.2, pp.286-291 (Engl. transl. in: J. Eng. Phys., 1980, v.39, No.2).

69. I.G.Kolker, A.Yu.Odinokov. Izv. Vyssh. Uchebn. Zaved., Aviatekhnika, 1988, No.2, pp.36-39 (in Russian).

70. V.G.Labeish. Liquid Cooling of High Temperature Metal. - Leningrad: Leningrad University, 1983, 172 p. (in Russian).

Third International Conference on Inverse Design Concepts and Optimization in Engineering Sciences (ICIDES-III), Editor: G.S. Dulikravich, Washington D.C., October 23-25, 1991.

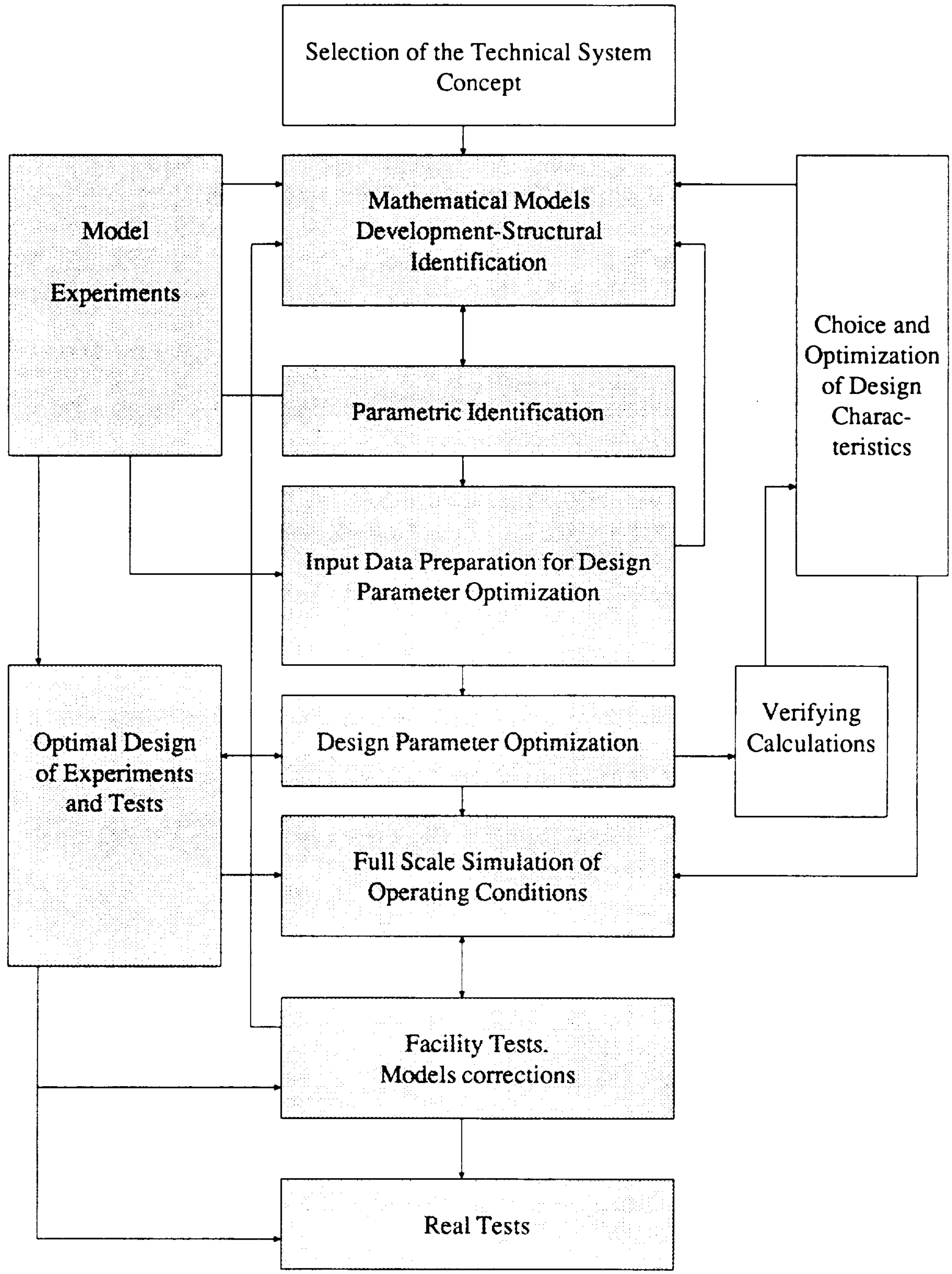

Fig. 1. Block Diagram of Design and Testing